Raymond Smullyan

What is the name of this book?
THE ISLAND OF KNIGHTS AND KNAVES

There is a wide variety of puzzles about an island in which certain inhabitants called "knights" always tell the truth, and others called "knaves" always lie. It is assumed that every inhabitant of the island is either a knight or a knave. I shall start with a well known puzzle of this type and then follow it with a variety of puzzles of my own.
26. According to this old problem, three of the inhabitants - A, B, and C - were standing together in a garden. A stranger passed by and asked A, "Are you a knight or a knave?" A answered, but rather indistinctly, so the stranger could not make out what he said. The stranger then asked B, "What did A say?" B replied, "A said that he is a knave." At this point the third man, C, said, "Don't believe B; he is lying!"

The question is, what are B and C?
solution

26. It is impossible for either a knight or a knave to say, "I'm a knave," because a knight wouldn't make the false statement that he is a knave, and a knave wouldn't make the true statement that he is a knave. Therefore A never did say that he was a knave. So B lied when he said that A said that he was a knave. Hence B is a knave. Since C said that B was lying and B was indeed lying, then C spoke the truth, hence is a knight. Thus B is a knave and C is a knight. (It is impossible to know what A is.)
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27. When I came upon the above problem, it immediately struck me that C did not really function in any essential way; he was sort of an appendage. That is to say, the moment B spoke, one could tell without C’s testimony that B was lying. The following variant of the problem eliminates that feature.

Suppose the stranger, instead of asking A what he is, asked A, "How many knights are among you?" Again A answers indistinctly. So the stranger asks B, "What did A say? B replies, "A said that there is one knight among us." Then C says, "Don't believe B; he is lying!"

The question is, what are B and C?
27. The answer is the same as that of the preceding problem, though the reasoning is a bit different. The first thing to observe is that B and C must be of opposite types, since B contradicts C. So of these two, one is a knight and the other a knave. Now, if A were a knight, then there would be two knights present, hence A would not have lied and said there was only one. On the other hand, if A were a knave, then it would be true that there was exactly one knight present; but then A, being a knave, couldn't have made that true statement. Therefore A could not have said that there was one knight among them. So B falsely reported A's statement, and thus B is a knave and C is a knight.
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28. In this problem, there are only two people, A and B, each of whom is either a knight or a knave. A makes the following statement: "At least one of us is a knave."

What are A and B?
solution

28. Suppose A were a knave. Then the statement "At least one of us is a knave" would be false (since knaves make false statements); hence they would both be knights. Thus, if A were a knave he would also have to be a knight, which is impossible. Therefore A is not a knave; he is a knight. Therefore his statement must be true, so at least one of them really is a knave. Since A is a knight, then B must be the knave. So A is a knight and B is a knave.
28. **Suppose** A were a knave. **Then** the statement "At least one of us is a knave" would be false (since knaves make false statements); **hence** they would both be knights. **Thus,** if A were a knave he would also have to be a knight, which is impossible. Therefore A is not a knave; he is a knight. Therefore his statement must be true, so at least one of them really is a knave. Since A is a knight, then B must be the knave. So A is a knight and B is a knave.
29. Suppose A says: “Either I am a Knave or B is a knight.” What are A and B?
30. Suppose A says, "Either I am a knave or else two plus two equals five." What would you conclude?
31. Again we have three people, A, B, C, each of whom is either a knight or a knave. A and B make the following statements:

A: All of us are knaves.
B: Exactly one of us is a knight.

What are A, B, C?
32. Suppose instead, A and B say the following:

A: All of us are knaves.

B: Exactly one of us is a knave.

Can it be determined what B is? Can it be determined what C is?
33. Suppose A says, "I am a knave, but B isn't." What are A and B?
34. We again have three inhabitants, A, B, and C, each of whom is a knight or a knave. Two people are said to be of the same type if they are both knights or both knaves. A and B make the following statements:

A: B is a knave.
B: A and C are of the same type.

What is C?
35. Again three people A, B, and C. A says "B and C are of the same type." Someone then asks C, "Are A and B of the same type?"

What does C answer?
36. An Adventure of Mine.

This is an unusual puzzle; moreover it is taken from real life. Once when I visited the island of knights and knaves, I came across two of the inhabitants resting under a tree. I asked one of them, "Is either of you a knight?" He responded, and I knew the answer to my question.

What is the person to whom I addressed the question? Is he a knight or a knave? And what is the other one? I can assure you, I have given you enough information to solve this problem.
37. Suppose you visit the island of knights and knaves. You come across two of the inhabitants lazily lying in the sun. You ask one of them whether the other one is a knight, and you get a (yes-or-no) answer. Then you ask the second one whether the first one is a knight. You get a (yes-or-no) answer.

Are the two answers necessarily the same?
38. This time you come across just one inhabitant lazily lying in the sun. You remember that his first name is either Edwin or Edward, but you cannot remember which. So you ask him his first name and he answers "Edward."

What is his first name?